Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

Let v_1=volume of gas at pressure P_1.
v_2=volume of gas at pressure P_2.
v=volume of gas at pressure P.
Work done by isothermic expansion from v_2 to v_1 is

$$w=\int_{v_2}^{v_1} Pdv.$$

But $P=P_2v_2/v$.

$$\therefore w=P_2v_2\int_{v_2}^{v_1}\frac{dv}{v}=P_2v_2\log(v_1/v_2).$$

The work done against gravity by lifting C cubic feet of water through an average height $\frac{1}{2}l$ is $W=\frac{1}{2}Cl\times62\frac{1}{2}=\frac{125}{4}Cl$.

Volume of water$=\frac{1}{4}\pi aD^2$, $v_1=\frac{1}{4}\pi D^2(L-a)$.

$$v_2=\frac{P_1v_1}{P_2}=\frac{\pi D^2P_1}{4P_2}(L-a).$$

$$C=v_1-v_2=\tfrac{1}{4}\pi D^2(L-a)\left(\frac{P_2-P_1}{P_2}\right).$$

$$l=\frac{(L-a)(P_2-P_1)}{P_2},\quad \frac{v_1}{v_2}=\frac{P_2}{P_1}.$$

$$\therefore w=\tfrac{1}{4}\pi D^2P_1(L-a)\log\left(\frac{P_2}{P_1}\right).$$

$$W=\tfrac{125}{16}\pi D^2(L-a)^2\left(\frac{P_2-P_1}{P_2}\right)^2.$$

Total work done$=w+W$.

Work done against pressure is the same as the work of expansion.

P_1 and P_2 are supposed to be given in pounds.

120. Proposed by W. J. GREENSTREET, M. A., Editor of The Mathematical Gazette, Stroud, Gloucestershire, England.

I project an elastic particle along a chord c of a smooth fixed circular rim of diameter d. The coefficient of elasticity between the particle and the rim is e, and the particle continually rebounds. Find the length of the chord described after the nth rebound.

Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

Let β, β_1, β_2, β_3, ...β_n be the angles the particle makes with the diameter before the first and after the first, second, third, and nth rebound, respectively, x=length of chord ofter the nth rebound.

Then $\cot\beta = c/\sqrt{(d^2-c^2)}$.

$\cot\beta_1 = e\cot\beta = ec/\sqrt{(d^2-c^2)}$.

$\cot\beta_2 = e\cot\beta_1 = e^2c/\sqrt{(d^2-c^2)}$.

. . . .

$\cot\beta_n = e^n c\sqrt{(d^2-c^2)} = x/\sqrt{(d^2-x^2)}$.

$\therefore\ xcde^n/\sqrt{[d^2-c^2(1-e^{2n})]}$.

If $e=1$, $x=c$.

AVERAGE AND PROBABILITY.

102. Proposed by PROFESSOR CAVALLIN.

A random straight line is determined by two points taken at random within a sphere; find the average velocity acquired by a particle in descending the line. [No. 6742, *Educational Times*. Unsolved.]

Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

Let (x, y, z), (u, v, w) be the coördinates of the two random points with center of sphere as origin. Let a=radius, $\sqrt{(a^2-x^2)}=y'$, $\sqrt{(a^2-u^2)}=v'$, $\sqrt{(a^2-x^2-y^2)}=z'$, $\sqrt{(a^2-u^2-v^2)}=w'$. The elevation of the one end of the line above the other$=(u-x)$.

Velocity$=\sqrt{[2g(u-x)]}$. The limits of x are $-a$ and a; of u, x and a and doubled for the case when $u<x$; of y, $-y'$ and y'; of z, $-z'$ and z'; of v, $-v'$ and v'; of w, $-w'$ and w'. Then since $(\frac{4}{3}\pi a^3)^2$ is the number of ways the two points can be taken, we get

$$\triangle = \frac{2}{(\frac{4}{3}\pi a^3)^2}\int_{-a}^{a}\int_{x}^{a}\int_{-y'}^{y'}\int_{-z'}^{z'}\int_{-v'}^{v'}\int_{-w'}^{w'}\sqrt{[2g(u-x)]}\,dxdudydzdvdw$$

$$=\frac{9\sqrt{2g}}{8a^6}\int_{-a}^{a}\int_{x}^{a}\sqrt{(u-x)}(a^2-x^2)(a^2-u^2)dxdu$$

$$=\frac{3\sqrt{2g}}{35a^6}\int_{-a}^{a}(5a+2x)(a^2-x^2)(a-x)^{\frac{5}{2}}\,dx$$

Let $x=a\cos\theta$.

$$\therefore\ \triangle = \frac{192\sqrt{ag}}{35}\int_0^{\pi}(7-4\sin^2\tfrac{1}{2}\theta)\sin^8\tfrac{1}{2}\theta\cos^3\tfrac{1}{2}\theta\,d\theta = \frac{256\sqrt{ag}}{273}.$$

103. Proposed by LON C. WALKER, A. M., Assistant Professor of Mathematics, Leland Stanford Jr. University, Palo Alto, Cal.

A circle is drawn at random both in magnitude and position, but so as to lie wholly on the surface of a given semi-circle. Show that the chance that a